CBSE 2025 EXAMINATIONS
XII MATHEMATICS (041)
Series W1XZY ♦ Q.P. Code 65/1/1; 65/1/2; 65/1/3

Time Allowed : 180 Minutes **Max. Marks : 80**

General Instructions :

1. This Question paper contains **five sections - A, B, C, D and E**. Each section is compulsory. However, there are **internal choices** in some questions.
2. Section A has **18 MCQs** and **02 Assertion-Reason (A-R)** based questions of **1 mark** each.
   Section B has **05 questions** of **2 marks** each.
   Section C has **06 questions** of **3 marks** each.
   Section D has **04 questions** of **5 marks** each.
   Section E has **03 Case-study / Source-based / Passage-based** questions with **sub-parts** (**4 marks** each).
3. There is no overall choice. However, **internal choice** has been provided in
   - **02 Questions of Section B**
   - **03 Questions of Section C**
   - **02 Questions of Section D**
   - **02 Questions of Section E**

   You have to attempt only one of the alternatives in all such questions.

☑ Q.P. Code - 65/1/1

SECTION A

This section has 20 multiple choice questions of 1 mark each.

01. If $A=\begin{bmatrix}-1&0&0\\0&1&0\\0&0&1\end{bmatrix}$, then A^{-1} is

(a) $\begin{bmatrix}-1&0&0\\0&-1&0\\0&0&-1\end{bmatrix}$ (b) $\begin{bmatrix}1&0&0\\0&-1&0\\0&0&-1\end{bmatrix}$ (c) $\begin{bmatrix}-1&0&0\\0&-1&0\\0&0&1\end{bmatrix}$ (d) $\begin{bmatrix}-1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

Sol. (d) Note that $\begin{bmatrix}a&0&0\\0&b&0\\0&0&c\end{bmatrix}^{-1}=\begin{bmatrix}a^{-1}&0&0\\0&b^{-1}&0\\0&0&c^{-1}\end{bmatrix}$; $a\neq 0, b\neq 0, c\neq 0$.

02. If vector $\vec{a}=3\hat{i}+2\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$, then which of the following is correct?

(a) $\vec{a}\parallel\vec{b}$ (b) $\vec{a}\perp\vec{b}$ (c) $|\vec{b}|>|\vec{a}|$ (d) $|\vec{a}|=|\vec{b}|$

Sol. (b) Note that, $\vec{a}.\vec{b}=0$.

03. $\int_{-1}^{1}\frac{|x|}{x}dx$, $x\neq 0$ is equal to

(a) -1 (b) 0 (c) 1 (d) 2

Sol. (b) Let $f(x)=\frac{|x|}{x}$. Then $f(-x)=\frac{|-x|}{-x}=-\frac{|x|}{x}=-f(x)$. So, $f(x)$ is an odd function.

Using $\int_{-a}^{a}f(x)dx=\begin{cases}0, \text{ if } f(x) \text{ is odd}\\ 2\int_{0}^{a}f(x)dx, \text{ if } f(x) \text{ is even}\end{cases}$, we get $\int_{-1}^{1}\frac{|x|}{x}dx=0$.

04. Which of the following is **not** a homogeneous function of x and y?

(a) $y^2 - xy$ (b) $x - 3y$ (c) $\sin^2\frac{y}{x} + \frac{y}{x}$ (d) $\tan x - \sec y$

Sol. (d) $\tan x - \sec y$ is not homogeneous function of x and y.

Recall that, if $f(kx, ky) = k^n f(x, y)$, then we say f is homogeneous function of degree n.

05. If $f(x) = |x| + |x-1|$, then which of the following is correct?

(a) $f(x)$ is both continuous and differentiable, at $x = 0$ and $x = 1$

(b) $f(x)$ is differentiable but not continuous, at $x = 0$ and $x = 1$

(c) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$

(d) $f(x)$ is neither continuous nor differentiable, at $x = 0$ and $x = 1$

Sol. (c) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$.

06. If A is square matrix of order 2 such that $\det(A) = 4$, then $\det(4\ \text{adj.}A)$ is equal to

(a) 16 (b) 64 (c) 256 (d) 512

Sol. (b) $|4\,\text{adj.}A| = 4^2\,|\text{adj.}A| = 16|A|^{2-1} = 16 \times 4 = 64$.

07. If E and F are two independent events such that $P(E) = \frac{2}{3}$, $P(F) = \frac{3}{7}$, then $P(E \mid \bar{F})$ is equal to

(a) $\frac{1}{6}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{7}{9}$

Sol. (c) $P(E \mid \bar{F}) = \frac{P(E \cap \bar{F})}{P(\bar{F})} = \frac{P(E).P(\bar{F})}{P(\bar{F})} = P(E) = \frac{2}{3}$.

As E and F are independent events so, E and $\bar{F}$ also independent events.

08. The absolute maximum value of function $f(x) = x^3 - 3x + 2$ in $[0, 2]$ is

(a) 0 (b) 2 (c) 4 (d) 5

Sol. (c) For $f(x) = x^3 - 3x + 2$, $f'(x) = 3x^2 - 3 = 3(x^2 - 1)$

When $f'(x) = 0$, then it gives $x = 1, -1$. Clearly, $x = 1 \in [0, 2]$.

Now $f(0) = 0^3 - 3\times 0 + 2 = 2$, $f(1) = 1^3 - 3\times 1 + 2 = 0$, $f(2) = 2^3 - 3\times 2 + 2 = 4$.

Hence, absolute maximum value of $f(x)$ is 4.

09. Let $A = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 4 & -1 \\ -3 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -2 \\ -5 \\ -7 \end{bmatrix}$, $C = \begin{bmatrix} 9 & 8 & 7 \end{bmatrix}$, which of the following is defined?

(a) Only AB (b) Only AC (c) Only BA (d) All AB, AC and BA

Sol. (a) Only AB is defined.

10. If $\int \frac{2^{\frac{1}{x}}}{x^2} dx = k.2^{\frac{1}{x}} + C$, then k is equal to

(a) $-\frac{1}{\log 2}$ (b) $-\log 2$ (c) -1 (d) $\frac{1}{2}$

Sol. (a) Put $2^{\frac{1}{x}} = t \Rightarrow \frac{2^{\frac{1}{x}}}{x^2} dx = \frac{-dt}{\log 2}$.

So, $\int \frac{2^{\frac{1}{x}}}{x^2} dx = \int \frac{-dt}{\log 2} = -\frac{t}{\log 2} + C = -\frac{1}{\log 2} \times 2^{\frac{1}{x}} + C \ldots (i)$

Given that $\int \frac{2^{\frac{1}{x}}}{x^2} dx = k.2^{\frac{1}{x}} + C$ …(ii)

By comparing (i) and (ii), we get $k = -\frac{1}{\log 2}$.

11. If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, $|\vec{a}|=\sqrt{37}$, $|\vec{b}|=3$, and $|\vec{c}|=4$, then angle between $\vec{b}$ and $\vec{c}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

Sol. (c) $\vec{a}+\vec{b}+\vec{c}=\vec{0} \Rightarrow \vec{b}+\vec{c}=-\vec{a}$

Consider $(\vec{b}+\vec{c}).(\vec{b}+\vec{c})=(-\vec{a}).(-\vec{a})$

$\Rightarrow |\vec{b}|^2+2\vec{b}.\vec{c}+|\vec{c}|^2=|\vec{a}|^2$

$\Rightarrow |\vec{b}|^2+2|\vec{b}||\vec{c}|\cos\theta+|\vec{c}|^2=|\vec{a}|^2$, where θ is the required angle

$\Rightarrow 9+2\times3\times4\cos\theta+16=37$

$\Rightarrow \cos\theta=\frac{1}{2} \quad \therefore \theta=\frac{\pi}{3}$.

12. The integrating factor of differential equation $(x+2y^3)\frac{dy}{dx}=2y$ is

(a) $e^{\frac{y^2}{2}}$ (b) $\frac{1}{\sqrt{y}}$ (c) $\frac{1}{y^2}$ (d) $e^{-\frac{1}{y^2}}$

Sol. (b) Rewriting the D.E., $\frac{dx}{dy}+\left(-\frac{1}{2y}\right)x=y^2$

Here $P(y)=-\frac{1}{2y}$. So, integrating factor will be $e^{\int -\frac{1}{2y}dy}=e^{-\frac{1}{2}\log y}=y^{-1/2}=\frac{1}{\sqrt{y}}$.

13. If $A=\begin{bmatrix}7 & 0 & x\\ 0 & 7 & 0\\ 0 & 0 & y\end{bmatrix}$ is a scalar matrix, then y^x is equal to

(a) 0 (b) 1 (c) 7 (d) ±7

Sol. (b) As the matrix A is a scalar matrix so, its all the non-diagonal elements must be zero (0) and all the diagonal elements must be same. That is, $y=7$, $x=0$. Hence, $y^x=7^0=1$.

14. The corner points of the feasible region in graphical representation of a L.P.P. are (2, 72), (15, 20) and (40, 15). If $Z=18x+9y$ be the objective function, then

(a) Z is maximum at (2, 72), minimum at (15, 20)
(b) Z is maximum at (15, 20), minimum at (40, 15)
(c) Z is maximum at (40, 15), minimum at (15, 20)
(d) Z is maximum at (40, 15), minimum at (2, 72)

Sol. (c) Here $Z=18x+9y=9(2x+y)$

Now $Z_{(2,72)}=9(4+72)=9(76)$, $Z_{(15,20)}=9(30+20)=9(50)$ and

$Z_{(40,15)}=9(80+15)=9(95)$.

Clearly, Z is maximum at (40, 15) and minimum at (15, 20).

15. If A and B are invertible matrices, then which of the following is **not** correct?

(a) $(A+B)^{-1}=B^{-1}+A^{-1}$ (b) $(AB)^{-1}=B^{-1}A^{-1}$

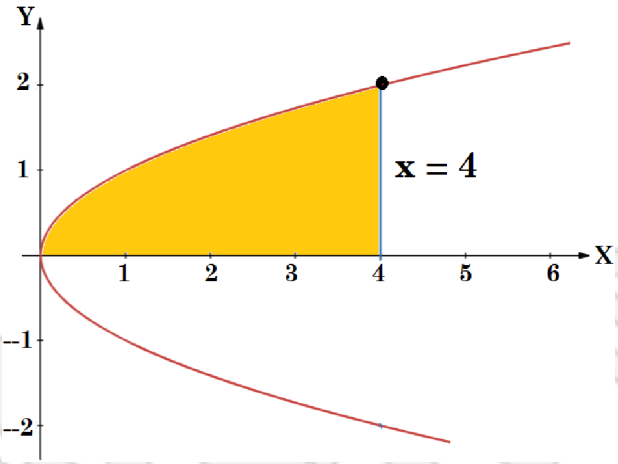
(c) $\text{adj.}(A) = |A|A^{-1}$ (d) $|A|^{-1} = |A^{-1}|$

Sol. (a) $(A+B)^{-1} = B^{-1} + A^{-1}$ is **not** correct.

16. If the feasible region of a linear programming problem with objective function $Z = ax + by$, is bounded, then which of the following is correct?
(a) It will only have a maximum value
(b) It will only have a minimum value
(c) It will have both maximum and minimum values
(d) It will have neither maximum nor minimum value

Sol. (c) It will have both maximum and minimum values.

17. The area of the shaded region bounded by the curves $y^2 = x$, $x = 4$ and the x-axis is given by



(a) $\int_0^4 x\,dx$

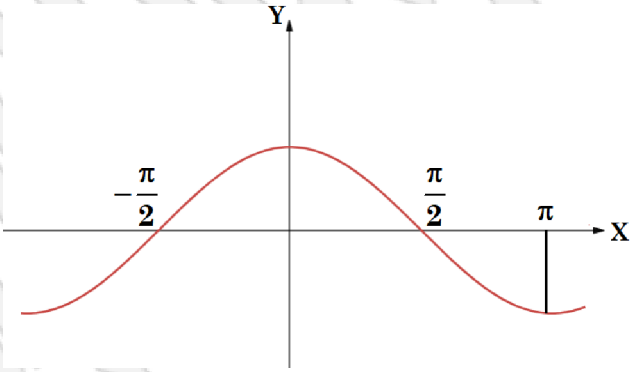
(b) $\int_0^2 y^2\,dy$

(c) $2\int_0^4 \sqrt{x}\,dx$

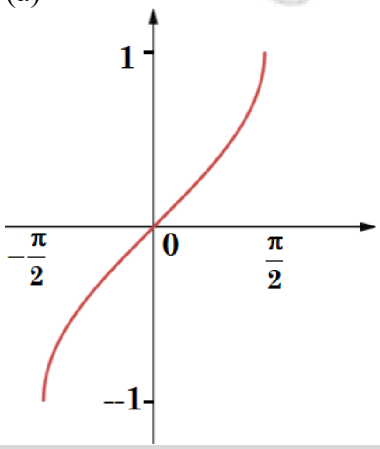
(d) $\int_0^4 \sqrt{x}\,dx$

Sol. (d) Area of the Shaded region $= \int_0^4 \sqrt{x}\,dx$.

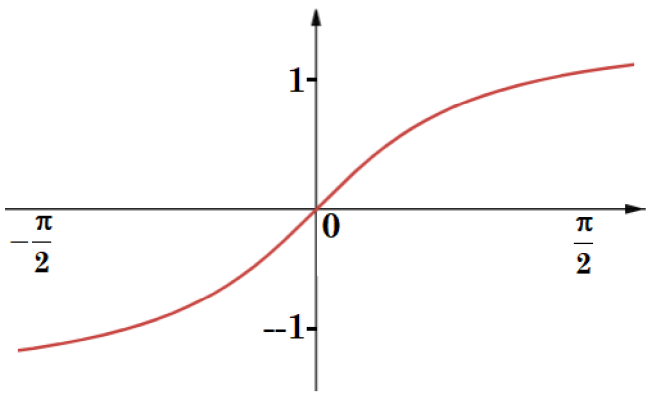
18. The graph of a trigonometric function is as shown. Which of the following will represent graph of its inverse?



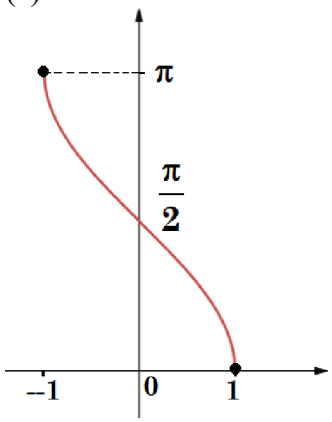
(a)



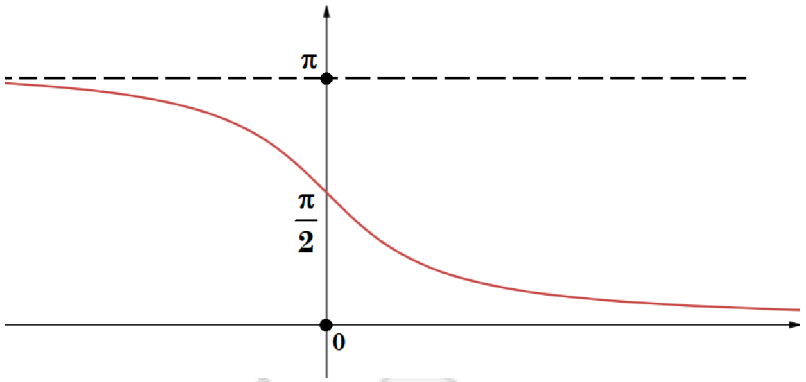
(b)



(c)



(d)



Sol. (c) The given graph is of the function $\cos x$. We have to select the option, where the graph of the inverse of $\cos x$ i.e., graph of $\cos^{-1} x$ is shown.

Remark The function $\cos x$ shown in the graph is **not** one-one. So, its inverse does not exist. To answer this question, we need to assume that inverse of given function is defined.

Direction : *Question numbers* ***19*** *and* ***20*** *are Assertion (A) and Reason (R) based questions carrying* ***1 mark*** *each. Two statements are given, one labeled Assertion (A) and other labeled Reason (R). Select the correct answer from the options given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true and Reason (R) is **not** the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

19. **Assertion (A) :** Let Z be the set of integers. A function $f: Z \to Z$ defined as $f(x) = 3x - 5$, $\forall\, x \in Z$ is a bijective.

Reason (R) : A function is a bijective if it is both surjective and injective.

Sol. (d) Assertion (A) is false but Reason (R) is true.

Note that, the function $f(x) = 3x - 5$ is injective.

Let $y = f(x)$, $y \in Z$.

When $y = 2 \in Z$, then $x = \frac{7}{3} \notin Z$. That is, f is not surjective, which means f is not bijective.

20. **Assertion (A) :** $f(x) = \begin{cases} 3x - 8, & x \le 5 \\ 2k, & x > 5 \end{cases}$ is continuous at $x = 5$ for $k = \frac{5}{2}$.

Reason (R) : For a function f to be continuous at $x = a$, $\lim\limits_{x \to a^-} f(x) = \lim\limits_{x \to a^+} f(x) = f(a)$.

Sol. (d) As f is continuous at $x = 5$ so, $\lim\limits_{x \to 5^+} f(x) = \lim\limits_{x \to 5^-} f(x) = f(5)$.

Taking $\lim\limits_{x \to 5^+} f(x) = f(5)$

$\Rightarrow \lim\limits_{x \to 5^+} (2k) = 3(5) - 8$

$\Rightarrow (2k) = 7 \quad \Rightarrow k = \frac{7}{2}$.

Clearly, Assertion (A) is false. Also, Reason (R) is true.

SECTION B

This section has 5 Very Short Answer questions of 2 marks each.

21. (a) Differentiate $2^{\cos^2 x}$ w.r.t. $\cos^2 x$.

OR

(b) If $\tan^{-1}(x^2+y^2)=a^2$, then find $\dfrac{dy}{dx}$.

Sol. (a) Let $y=2^{\cos^2 x}$, $z=\cos^2 x$.

Then $y=2^z$

On differentiating w.r.t. z, we get $\dfrac{dy}{dz}=2^z\times\log 2=2^{\cos^2 x}\times\log 2$.

OR

(b) Given $\tan^{-1}(x^2+y^2)=a^2 \quad\Rightarrow (x^2+y^2)=\tan a^2$

On differentiating w.r.t. x, we get $2x+2y\times\dfrac{dy}{dx}=0 \quad\Rightarrow 2y\times\dfrac{dy}{dx}=-2x$

$\therefore \dfrac{dy}{dx}=-\dfrac{x}{y}$.

22. Evaluate $\tan^{-1}\left[2\sin\left(2\cos^{-1}\dfrac{\sqrt{3}}{2}\right)\right]$.

Sol. $\tan^{-1}\left[2\sin\left(2\cos^{-1}\dfrac{\sqrt{3}}{2}\right)\right]=\tan^{-1}\left[2\sin\left(2\times\dfrac{\pi}{6}\right)\right]=\tan^{-1}\left[2\sin\left(\dfrac{\pi}{3}\right)\right]$

$=\tan^{-1}\left[2\times\dfrac{\sqrt{3}}{2}\right]=\tan^{-1}\left[\sqrt{3}\right]=\dfrac{\pi}{3}$.

23. The diagonals of a parallelogram are given by $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+3\hat{j}-\hat{k}$. Find the area of the parallelogram.

Sol. Note that, $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&-1&1\\1&3&-1\end{vmatrix}=-2\hat{i}+3\hat{j}+7\hat{k}$.

Now, the area of parallelogram $=\dfrac{1}{2}\left|\vec{a}\times\vec{b}\right|=\dfrac{1}{2}\sqrt{(-2)^2+3^2+7^2}=\dfrac{\sqrt{62}}{2}$ Sq.units.

24. Find the intervals in which $f(x)=5x^{\frac{3}{2}}-3x^{\frac{5}{2}}$ is (i) increasing (ii) decreasing.

Sol. The domain of given function $f(x)$ is $x\in[0,\infty)$.

Now $f'(x)=5\times\dfrac{3}{2}x^{\frac{1}{2}}-3\times\dfrac{5}{2}x^{\frac{3}{2}}=\dfrac{15}{2}\sqrt{x}\,(1-x)$.

For $f'(x)=0$, $\dfrac{15}{2}\sqrt{x}\,(1-x)=0$, we get $x=0,1$.

(i) In $x\in[0,1]$, $f'(x)\geq 0$. So, $f(x)$ is increasing in $x\in[0,1]$.

(ii) In $x\in[1,\infty)$, $f'(x)\leq 0$. So, $f(x)$ is decreasing in $x\in[1,\infty)$.

25. (a) Two friends while flying kites from different locations, find the strings of their kites crossing each other. The strings can be represented by the vectors $\vec{a}=3\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}-2\hat{j}+4\hat{k}$. Determine the angle formed between the kite strings. Assume there is no slack in the strings.

OR

(b) Find a vector of magnitude 21 units in the direction opposite to that of $\overrightarrow{AB}$, where A are B are the points A(2, 1, 3) and B(8, –1, 0) respectively.

Sol. (a) Let the required angle between the kite strings be θ.

Then $\cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$

$$\Rightarrow \cos\theta = \frac{(3\hat{i}+\hat{j}+2\hat{k}).(2\hat{i}-2\hat{j}+4\hat{k})}{\sqrt{9+1+4}\sqrt{4+4+16}} = \frac{6-2+8}{\sqrt{14}\sqrt{24}} = \frac{12}{\sqrt{14}\times 2\sqrt{6}} = \sqrt{\frac{6}{14}} = \sqrt{\frac{3}{7}}$$

$$\Rightarrow \theta = \cos^{-1}\sqrt{\frac{3}{7}} \text{ or, } \theta = \cos^{-1}\frac{\sqrt{21}}{7}.$$

OR

(b) Here $\overrightarrow{AB} = (8\hat{i}-\hat{j})-(2\hat{i}+\hat{j}+3\hat{k}) = 6\hat{i}-2\hat{j}-3\hat{k}$.

The vector which is in the direction opposite to that of $\overrightarrow{AB} = -\overrightarrow{AB} = -(6\hat{i}-2\hat{j}-3\hat{k})$.

$$\text{Required vector} = 21\times\left(-\frac{6\hat{i}-2\hat{j}-3\hat{k}}{\sqrt{36+4+9}}\right) = -\frac{21(6\hat{i}-2\hat{j}-3\hat{k})}{7} = -18\hat{i}+6\hat{j}+9\hat{k}.$$

SECTION C

There are 6 Short Answer questions in this section. Each is of 3 marks.

26. The side of an equilateral triangle is increasing at the rate of 3 cm/s. At what rate is its area increasing when the side of the triangle is 15 cm?

Sol. Let 'a' denote the side length of the triangle so, $\dfrac{da}{dt} = 3$ cm/s.

Since area of the equilateral triangle is, $A = \dfrac{\sqrt{3}\,a^2}{4}$

$$\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}\,a}{2}\times\frac{da}{dt}$$

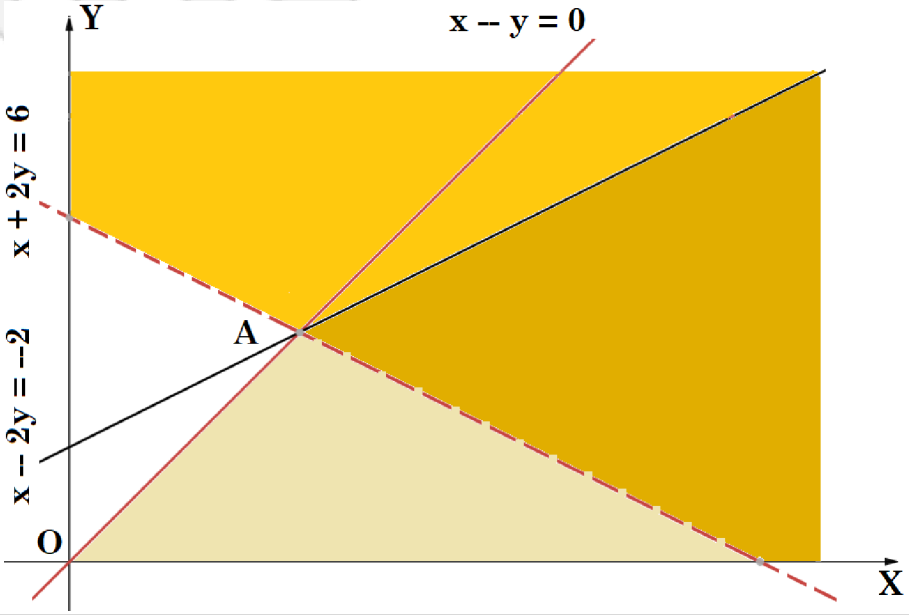
$$\therefore \left.\frac{dA}{dt}\right]_{\text{at } a=15\text{ cm}} = \frac{\sqrt{3}\times 15}{2}\times 3 = \frac{45\sqrt{3}}{2} \text{ cm}^2/\text{s}.$$

27. Solve the following linear programming problem graphically.

Maximise : $Z = x + 2y$

Subject to the constraints : $x - y \geq 0$, $x - 2y \geq -2$, $x \geq 0$, $y \geq 0$.

Sol. Consider the graph shown.



The corner points of feasible region are $O(0, 0)$ and $A(2, 2)$.

Corner Points	Value of Z
$O(0, 0)$	$Z_O = 0 + 2(0) = 0$
$A(2, 2)$	$Z_A = 2 + 2(2) = 6 \leftarrow$ Maximum

As the feasible region is unbounded so, $Z = 6$ may or may not be the maximum value of Z.
To check, let $x + 2y > 6$.
Note that, the feasible region of LPP and $x + 2y > 6$ have common points / common region.
So, there is **no** maximum value of Z.

28. (a) Find $\int \frac{x + \sin x}{1 + \cos x} dx$.

OR

(b) Evaluate $\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^3 x \sqrt{2 \sin 2x}}$.

Sol. (a) Let $I = \int \frac{x + \sin x}{1 + \cos x} dx$

$$\Rightarrow I = \int \frac{x}{1 + \cos x} dx + \int \frac{\sin x}{1 + \cos x} dx$$

$$\Rightarrow I = \int \frac{x}{2\cos^2 \frac{x}{2}} dx + \int \frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{2\cos^2 \frac{x}{2}} dx$$

$$\Rightarrow I = \int x \left(\frac{1}{2} \sec^2 \frac{x}{2} \right) dx + \int \tan \frac{x}{2} dx$$

Applying By Parts in the first integral, we get

$$I = x \int \left(\frac{1}{2} \sec^2 \frac{x}{2} \right) dx - \int \left[\frac{d}{dx}(x) \int \left(\frac{1}{2} \sec^2 \frac{x}{2} \right) dx \right] dx + \int \tan \frac{x}{2} dx$$

$$\Rightarrow I = x \left(\frac{1}{2} \times 2 \tan \frac{x}{2} \right) - \int \left[1 \times \left(\frac{1}{2} \times 2 \tan \frac{x}{2} \right) \right] dx + \int \tan \frac{x}{2} dx$$

$$\Rightarrow I = x \left(\tan \frac{x}{2} \right) - \int \tan \frac{x}{2} dx + \int \tan \frac{x}{2} dx$$

$$\therefore I = x \tan \frac{x}{2} + C.$$

OR

(b) Let $I = \int_0^{\pi/4} \frac{dx}{\cos^3 x \sqrt{2 \sin 2x}}$

Divide and multiply Dr by $\cos x$, we get $I = \int_0^{\pi/4} \frac{dx}{\frac{\cos^4 x \sqrt{2 \sin 2x}}{\cos x}} = \int_0^{\pi/4} \frac{\sec^4 x \, dx}{\frac{\sqrt{2 \sin 2x}}{\cos x}}$

$$\Rightarrow I = \int_0^{\pi/4} \frac{(1 + \tan^2 x) \sec^2 x \, dx}{\sqrt{\frac{4 \sin x \cos x}{\cos^2 x}}}$$

$$\Rightarrow I=\int_0^{\pi/4}\frac{(1+\tan^2 x)\sec^2 x\,dx}{2\sqrt{\tan x}}$$

Put $\tan x=t\Rightarrow \sec^2 x dx=dt$. When $x=0\Rightarrow t=0$; when $x=\frac{\pi}{4}\Rightarrow t=1$.

$$\therefore I=\frac{1}{2}\int_0^1\frac{(1+t^2)dt}{\sqrt{t}}$$

$$\Rightarrow I=\frac{1}{2}\int_0^1\left(\frac{1}{\sqrt{t}}+t^{3/2}\right)dt$$

$$\Rightarrow I=\frac{1}{2}\left[2\sqrt{t}+\frac{2}{5}t^{5/2}\right]_0^1$$

$$\Rightarrow I=\left[\sqrt{t}+\frac{1}{5}t^{5/2}\right]_0^1$$

$$\Rightarrow I=\left[\sqrt{1}+\frac{1}{5}\times 1^{5/2}\right]-\left[\sqrt{0}+\frac{1}{5}\times 0^{5/2}\right]$$

$$\therefore I=\frac{6}{5}.$$

29. (a) Verify that lines given by $\vec{r}=(1-\lambda)\hat{i}+(\lambda-2)\hat{j}+(3-2\lambda)\hat{k}$ and $\vec{r}=(\mu+1)\hat{i}+(2\mu-1)\hat{j}-(2\mu+1)\hat{k}$ are skew-lines. Hence, find shortest distance between the lines.

OR

(b) During a cricket match, the position of the bowler, the wicket keeper and the leg slip fielder are in a line given by $\vec{B}=2\hat{i}+8\hat{j}$, $\vec{W}=6\hat{i}+12\hat{j}$ and $\vec{F}=12\hat{i}+18\hat{j}$ respectively. Calculate the ratio in which the wicket keeper divides the line segment joining the bowler and the leg slip fielder.

Sol. (a) Rewriting the lines, we get

$\vec{r}=\hat{i}-2\hat{j}+3\hat{k}+\lambda(-\hat{i}+\hat{j}-2\hat{k})$ ...(1) and $\vec{r}=\hat{i}-\hat{j}-\hat{k}+\mu(\hat{i}+2\hat{j}-2\hat{k})$ ...(2)

Here $\vec{a}_1=\hat{i}-2\hat{j}+3\hat{k}$, $\vec{b}_1=-\hat{i}+\hat{j}-2\hat{k}$; $\vec{a}_2=\hat{i}-\hat{j}-\hat{k}$, $\vec{b}_2=\hat{i}+2\hat{j}-2\hat{k}$.

The direction ratios of both the lines (1) and (2) are –1, 1, –2 and 1, 2, –2 respectively.

Note that the d.r.'s of given lines are not proportional so, they are not parallel lines.

The lines will be skew if, apart from being non-parallel, they do not intersect each other.

Now $\vec{a}_2-\vec{a}_1=\hat{j}-4\hat{k}$, $\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-1&1&-2\\1&2&-2\end{vmatrix}=2\hat{i}-4\hat{j}-3\hat{k}$.

Shortest distance between the lines, $S.D.=\frac{|(\vec{b}_1\times\vec{b}_2).(\vec{a}_2-\vec{a}_1)|}{|\vec{b}_1\times\vec{b}_2|}$

$$\Rightarrow S.D.=\frac{|(2\hat{i}-4\hat{j}-3\hat{k}).(\hat{j}-4\hat{k})|}{\sqrt{4+16+9}}$$

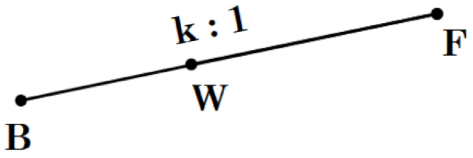
$$\Rightarrow S.D.=\frac{|-4+12|}{\sqrt{29}}=\frac{8}{\sqrt{29}}\text{ units}.$$

As $S.D.\neq 0$ so, the lines don't intersect. Hence, the lines are skew lines as they are non parallel.

OR

(b) Let $\vec{W}$ divides the line segment joining $\vec{B}$ and $\vec{F}$ in $k:1$.

Therefore, $\overrightarrow{W}=\dfrac{k.\vec{F}+1.\vec{B}}{k+1}$



$\Rightarrow 6\hat{i}+12\hat{j}=\dfrac{k.(12\hat{i}+18\hat{j})+1.(2\hat{i}+8\hat{j})}{k+1}$

$\Rightarrow 6\hat{i}+12\hat{j}=\left(\dfrac{12k+2}{k+1}\right)\hat{i}+\left(\dfrac{18k+8}{k+1}\right)\hat{j}$

By comparing the coefficients of $\hat{i}$ and $\hat{j}$, we get $6=\left(\dfrac{12k+2}{k+1}\right), 12=\left(\dfrac{18k+8}{k+1}\right)$

$\Rightarrow 6k+6=12k+2,\ 12k+12=18k+8$

$\Rightarrow 4=6k,\ 4=6k \qquad \Rightarrow k=\dfrac{4}{6}=\dfrac{2}{3}.$

Hence, the required ratio is $\dfrac{2}{3}:1$ i.e., $2:3$.

30. (a) The probability distribution for the number of students being absent in a class on a Saturday is as follows.

X	0	2	4	5
P(X)	p	2p	3p	p

where X is the number of students absent.

(i) Calculate p.

(ii) Calculate the mean of the number of absent students on Saturday.

OR

(b) For the vacancy advertised in the newspaper, 3000 candidates submitted their applications. From the data it was revealed that two third of the total applicants were females and other were males. The selection for the job was done through a written test. The performance of the applicants indicates that the probability of a male getting a distinction in written test is 0.4 and that a female getting a distinction is 0.35. Find the probability that the candidate chosen at random will have a distinction in the written test.

Sol. (a) (i) Since $\sum P(X)=1$ so, $p+2p+3p+p=1 \qquad \Rightarrow p=\dfrac{1}{7}.$

(ii) Mean $=\sum X.P(X)=0\times p+2\times 2p+4\times 3p+5\times p=21\times p=21\times\dfrac{1}{7}=3.$

OR

(b) Let E : the candidate chosen will have distinction in the written test.

Also let E_1 and E_2 be the event for male and female applicants respectively.

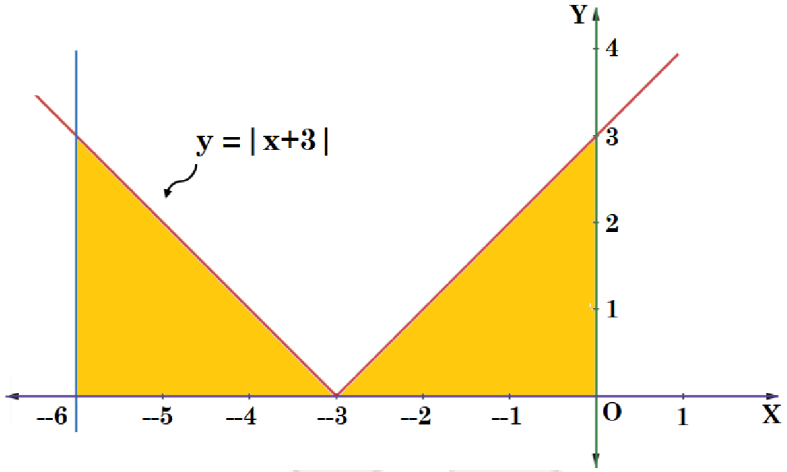
Now $P(E_1)=\dfrac{1}{3}, P(E_2)=\dfrac{2}{3}, P(E\,|\,E_1)=0.4, P(E\,|\,E_2)=0.35.$

$\therefore P(E)=P(E_1)\times P(E\,|\,E_1)+P(E_2)\times P(E\,|\,E_2)$

$\Rightarrow P(E)=\dfrac{1}{3}\times 0.4+\dfrac{2}{3}\times 0.35=\dfrac{1}{3}[0.4+0.70]=\dfrac{1.1}{3}=\dfrac{11}{30}.$

31. Sketch the graph of $y=|x+3|$ and find the area of the region enclosed by the curve, x-axis, between $x=-6$ and $x=0$, using integration.

Sol. We have $y = |x+3| = \begin{cases} x+3, \text{ if } x \geq -3 \\ -x-3, \text{ if } x < -3 \end{cases}$.



$$\text{Required area} = \int_{-6}^{0} y\,dx$$

$$= \int_{-6}^{-3} y\,dx + \int_{-3}^{0} y\,dx$$

$$= \int_{-6}^{-3} -(x+3)\,dx + \int_{-3}^{0} (x+3)\,dx$$

$$= -\frac{1}{2}\left[(x+3)^2\right]_{-6}^{-3} + \frac{1}{2}\left[(x+3)^2\right]_{-3}^{0}$$

$$= -\frac{1}{2}[0-9] + \frac{1}{2}[9-0]$$

$$= \frac{9}{2} + \frac{9}{2} = 9$$

Hence, the required area $= 9$ Sq. units.

SECTION D

There are 4 Long Answer questions in this section. Each is of 5 marks.

32. (a) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

OR

(b) If $x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$ and $y = \sin\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

Sol. (a) Put $x = \sin\alpha, y = \sin\beta \Rightarrow \alpha = \sin^{-1}x, \beta = \sin^{-1}y \ldots(i)$

$$\therefore \sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$$

$$\Rightarrow \sqrt{1-\sin^2\alpha} + \sqrt{1-\sin^2\beta} = a(\sin\alpha - \sin\beta)$$

$$\Rightarrow \cos\alpha + \cos\beta = 2a\left(\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}\right)$$

$$\Rightarrow 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = 2a\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\Rightarrow \cot\frac{\alpha-\beta}{2} = a$$

$$\Rightarrow \alpha - \beta = 2\cot^{-1}a$$

$$\Rightarrow \sin^{-1}x - \sin^{-1}y = 2\cot^{-1}a \qquad \text{[Using (i)}$$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}} \times \frac{dy}{dx} = 0$$

$$\therefore \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}.$$

OR

(b) We have $x = a\left\{\cos\theta + \log\tan\left(\frac{\theta}{2}\right)\right\}$, $y = \sin\theta$

Differentiating w.r.t. θ both the sides, we get

$$\frac{dx}{d\theta}=a\left\{-\sin\theta+\frac{1}{\tan\left(\frac{\theta}{2}\right)}\times\sec^2\left(\frac{\theta}{2}\right)\times\frac{1}{2}\right\},\ \frac{dy}{d\theta}=\cos\theta$$

$$\Rightarrow\frac{dx}{d\theta}=a\left\{-\sin\theta+\frac{1}{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}\right\}=a\left\{-\sin\theta+\frac{1}{\sin\theta}\right\}=a\left\{\frac{1-\sin^2\theta}{\sin\theta}\right\}=a\cot\theta\cos\theta$$

Now, $\frac{dy}{dx}=\left(\frac{dy}{d\theta}\right)\times\left(\frac{d\theta}{dx}\right)=\cos\theta\times\frac{1}{a\cos\theta\cot\theta}=\frac{\tan\theta}{a}$

Differentiating w.r.t. x both sides, we get $\frac{d^2y}{dx^2}=\frac{\sec^2\theta}{a}\times\frac{d\theta}{dx}$

$$\Rightarrow\ \frac{d^2y}{dx^2}=\frac{\sec^2\theta}{a}\times\frac{1}{a\cot\theta\cos\theta}=\frac{\sec^3\theta\tan\theta}{a^2}$$

So, $\left.\frac{d^2y}{dx^2}\right]_{\text{at }\theta=\pi/4}=\frac{\sec^3\frac{\pi}{4}\times\tan\frac{\pi}{4}}{a^2}=\frac{\left[\sqrt{2}\right]^3\times1}{a^2}=\frac{2\sqrt{2}}{a^2}$.

33. Find the absolute maximum and absolute minimum of function $f(x)=2x^3-15x^2+36x+1$ on $[1, 5]$.

Sol. For $f(x)=2x^3-15x^2+36x+1$, $f'(x)=6x^2-30x+36=6(x^2-5x+6)=6(x-2)(x-3)$

For $f'(x)=0$, $6(x-2)(x-3)=0$

$\Rightarrow x=2, 3\in[1, 5]$

Now $f(1)=2\times1^3-15\times1^2+36\times1+1=2-15+36+1=24$,

$f(2)=2\times2^3-15\times2^2+36\times2+1=16-60+72+1=29$,

$f(3)=2\times3^3-15\times3^2+36\times3+1=54-135+108+1=28$,

$f(5)=2\times5^3-15\times5^2+36\times5+1=250-375+180+1=56$.

Hence, the absolute maximum value of function is 56 and the absolute minimum value is 24.

34. (a) Find the image A' of the point A(1, 6, 3) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$.

Also, find the equation of the line joining A and A'.

OR

(b) Find a point P on the line $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}$ such that its distance from point Q(2, 4, –1) is 7 units. Also, find the equation of line joining P and Q.

Sol. (a) Given A(1, 6, 3). Let M be foot of perpendicular on the given line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}=\lambda$.

Any arbitrary point on the line is $M(\lambda, 2\lambda+1, 3\lambda+2)$.

Direction ratios of AM : $\lambda-1, 2\lambda-5, 3\lambda-1$.

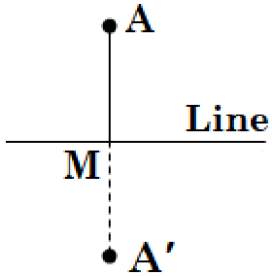
Since AM is perpendicular to the given line.

So by using $a_1a_2+b_1b_2+c_1c_2=0$ for $\perp$ lines, we get

$1(\lambda-1)+2(2\lambda-5)+3(3\lambda-1)=0$

$\Rightarrow\lambda-1+4\lambda-10+9\lambda-3=0$

$\Rightarrow\lambda=1$



$\therefore$ M(1, 3, 5) is the **foot of perpendicular** of the point A in line.

Let image of point A in the line be $A'(h, p, s)$.

Since M is the mid-point of AA', so $M\left(\frac{1+h}{2}, \frac{6+p}{2}, \frac{3+s}{2}\right) = M(1, 3, 5)$.

On comparing the respective coefficients, we obtain : $A'(1, 0, 7)$ as the **image** of A in the line.

The direction ratios of line AA' are $1-1, 0-6, 7-3$ i.e., $0, -6, 4$ i.e., $0, -3, 2$.

Now the equation of line joining A and A' is $\frac{x-1}{0} = \frac{y-6}{-3} = \frac{z-3}{2}$.

OR

(b) Let $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9} = \lambda$.

Any random point on the line will be given by $P(\lambda-5, 4\lambda-3, -9\lambda+6)$.

Since $PQ = 7$ so, $\sqrt{(\lambda-5-2)^2+(4\lambda-3-4)^2+(-9\lambda+6+1)^2} = 7$

$\Rightarrow (\lambda-7)^2+(4\lambda-7)^2+(-9\lambda+7)^2 = 49$

$\Rightarrow \lambda^2-14\lambda+49+16\lambda^2-56\lambda+49+81\lambda^2-126\lambda+49 = 49$

$\Rightarrow 98\lambda^2-196\lambda+98 = 0 \quad \Rightarrow \lambda^2-2\lambda+1 = 0 \quad \Rightarrow (\lambda-1)^2 = 0 \quad \therefore \lambda = 1$

Hence, the required point is $P(-4, 1, -3)$.

The d.r.'s of line joining P and Q is $2-(-4), 4-1, -1-(-3)$ i.e., $6, 3, 2$.

Hence, the equation of line joining P and Q is $\frac{x+4}{6} = \frac{y-1}{3} = \frac{z+3}{2}$.

35. A school wants to allocate students into three clubs : Sports, Music and Drama, under following conditions.

- The number of students in Sports club should be equal to the sum of the number of students in Music and Drama club.
- The number of students in Music club should be 20 more than half the number of students in Sports club.
- The total number of students to be allocated in all three clubs are 180.

Find the number of students allocated to different clubs, using matrix method.

Sol. Let x, y and z be the no. of students allocated to Sports, Music and Drama clubs respectively.

So, $x = y+z$, $y = \frac{x}{2}+20$, $x+y+z = 180$ i.e., $x-y-z = 0$, $x-2y = -40$, $x+y+z = 180$.

Let $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ -40 \\ 180 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

Clearly, $AX = B$ i.e., $X = A^{-1}B$ ...(i)

Now $|A| = 1(-2)+1(1)+(-1)(3) = -2+1-3 = -4 \neq 0 \quad \therefore A^{-1}$ exists.

Adjoint of matrix A, $\text{adj.}A = \begin{bmatrix} -2 & 0 & -2 \\ -1 & 2 & -1 \\ 3 & -2 & -1 \end{bmatrix}$.

Inverse of matrix A, $A^{-1} = \frac{\text{adj.}A}{|A|} = \frac{1}{4}\begin{bmatrix} 2 & 0 & 2 \\ 1 & -2 & 1 \\ -3 & 2 & 1 \end{bmatrix}$.

By (i), $X = \frac{1}{4}\begin{bmatrix} 2 & 0 & 2 \\ 1 & -2 & 1 \\ -3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ -40 \\ 180 \end{bmatrix} \Rightarrow X = \frac{1}{4}\begin{bmatrix} 360 \\ 260 \\ 100 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 90 \\ 65 \\ 25 \end{bmatrix}$

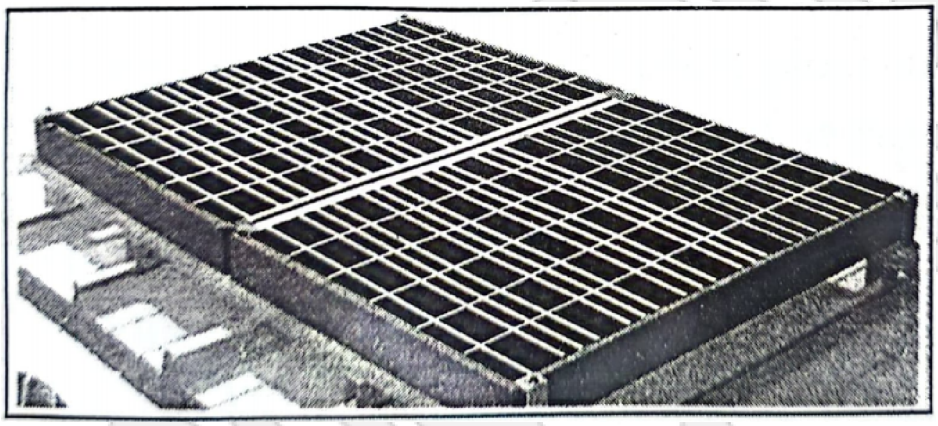
By equality of matrices, we get $x = 90,\ y = 65,\ z = 25$.

Hence, the number of students allocated in Sports club : 90, the number of students allocated in Music club : 65 and, the number of students allocated in Drama club : 25.

SECTION E

In this section there are 3 Case Study questions of 4 marks each.

36. A technical company is designing a rectangular solar panel installation on a roof using 300 metres of boundary material. The design includes a partition running parallel to one of the sides dividing the area (roof) into two sections.



Let the length of the side perpendicular to the partition be x metres and with parallel to the partition be y metres.

Based on this information, answer the following questions.

(i) Write the equation for the total boundary material used in the boundary and parallel to the partition in terms of x and y.

(ii) Write the area of the solar panel as a function of x.

(iii) (a) Find the critical points of the area function. Use second derivative test to determine critical points at the maximum area. Also, find the maximum area.

OR

(iii) (b) Using first derivative test, calculate the maximum area the company can enclose with the 300 metres of boundary material, considering the parallel partition.

Sol. **(i)** $2(x + y) = 300$ i.e., $x + y = 150$.

(ii) Area, $A = xy = x(150 - x)$.

(iii) (a) We have $A = x(150 - x) = 150x - x^2$

$\Rightarrow \frac{dA}{dx} = 150 - 2x$ and $\frac{d^2A}{dx^2} = -2$.

For critical points, $\frac{dA}{dx} = 0,\ 150 - 2x = 0$

$\Rightarrow x = \frac{150}{2} = 75$

Note that, $\left(\frac{d^2A}{dx^2}\right)_{\text{at } x=75} = -2 < 0$. So, A is maximum at $x = 75$.

Also maximum area is, $A = 75(150-75) = 75\times 75 = 5625 \text{ m}^2$.

OR

(iii) (b) We have $A = x(150-x) = 150x - x^2$

$\Rightarrow \frac{dA}{dx} = 150 - 2x$.

For critical points, $\frac{dA}{dx} = 0, 150 - 2x = 0 \quad \Rightarrow x = \frac{150}{2} = 75$

As $\frac{dA}{dx}$ changes its sign from positive to negative as x passes through $x = 75$ from left to right.

That means, $x = 75$ is the point of maximum.

Hence, maximum value of area, $A = 75(150-75) = 75\times 75 = 5625 \text{ m}^2$.

Remark Since diagram / language in this question is unclear so, students may have solved this problem by taking $2x + 2y = 300$ OR $2x + 4y = 300$ OR $2x + 3y = 300$ OR $4x + 4y = 300$. The solution of sub-parts will differ accordingly.

37. A class-room teacher is keen to assess the learning of her students on the concept of "relations" taught to them. She writes the following five relations each defined on the set $A = \{1, 2, 3\}$.

$R_1 = \{(2, 3), (3, 2)\}$

$R_2 = \{(1, 2), (1, 3), (3, 2)\}$

$R_3 = \{(1, 2), (2, 1), (1, 1)\}$

$R_4 = \{(1, 1), (1, 2), (3, 3), (2, 2)\}$

$R_5 = \{(1, 1), (1, 2), (3, 3), (2, 2), (2, 1), (2, 3), (3, 2)\}$

The students are asked to answer the following questions about the above relations.

(i) Identify the relation which is reflexive, transitive but not symmetric.

(ii) Identify the relation which is reflexive and symmetric but not transitive.

(iii) (a) Identify the relations which are symmetric but neither reflexive nor transitive.

OR

(iii) (b) What pairs should be added to the relation R_2 to make it an equivalence relation?

Sol. **(i)** Note that, $(1, 2) \in R_4$ but $(2, 1) \notin R_4$. So, it is not symmetric relation. However it is reflexive and transitive.

(ii) Note that, $(1, 2) \in R_5$ and $(2, 3) \in R_5$ but $(1, 3) \notin R_5$. So, it is not transitive. However it is reflexive and symmetric.

(iii) (a) Note that, $(2, 3) \in R_1$ and $(3, 2) \in R_1$. So, it is symmetric. However it is not reflexive as $(a, a) \notin R_1 \ \forall\ a \in A$. Also, it is not transitive as well since, $(2, 2) \notin R_1$ and $(3, 3) \notin R_1$.

Further $(a, a) \notin R_3 \ \forall\ a \in A$. So, R_3 is not reflexive.

Also R_3 is not transitive as $(2, 1) \in R_3$ and $(1, 2) \in R_3$ but, $(2, 2) \notin R_3$.

However R_3 is symmetric as $(1, 2) \in R_3$ and $(2, 1) \in R_3$.

Hence, R_1 and R_3 are the relations which are symmetric but neither reflexive nor transitive.

OR

(iii) (b) To make R_2 reflexive, we must add $(1, 1), (2, 2), (3, 3)$ in it.

Also, to make it symmetric, we must add $(2, 1), (3, 1), (2, 3)$.

Therefore, once we add the pairs $(1,1),(2,2),(3,3),(2,1),(3,1)$ and $(2,3)$ in R_2, then it will become reflexive, symmetric and transitive. Hence, R_2 will be equivalence relation.

38. A bank offers loan to its customers on different types of interest namely, fixed rate, floating rate and variable rate.



From the past data with the bank, it is known that a customer avails loan on fixed rate, floating rate or variable rate with probabilities 10%, 20% and 70% respectively. A customer after availing loan can pay the loan or default on loan repayment. The bank data suggests that the probability that a person defaults on loan after availing it at fixed rate, floating rate and variable rate is 5%, 3% and 1% respectively.

Based on the above information, answer the following questions.

(i) What is the probability that a customer after availing the loan will default on the loan repayment?

(ii) A customer after availing the loan, defaults on loan repayment. What is the probability that he availed the loan at a variable rate of interest?

Sol. Let E_1, E_2 and E_3 be the events that a customer avails loan on fixed rate, floating rate and variable rate respectively. Also let E be the event that the person defaults on the loan.

Now $P(E_1)=10\%=\frac{1}{10}$, $P(E_2)=20\%=\frac{2}{10}$, $P(E_3)=70\%=\frac{7}{10}$.

Also $P(E\,|\,E_1)=5\%$, $P(E\,|\,E_2)=3\%$, $P(E\,|\,E_3)=1\%$.

(i) $P(E)=P(E_1).P(E\,|\,E_1)+P(E_2).P(E\,|\,E_2)+P(E_3).P(E\,|\,E_3)$

$$\Rightarrow P(E)=\frac{1}{10}\times\frac{5}{100}+\frac{2}{10}\times\frac{3}{100}+\frac{7}{10}\times\frac{1}{100}$$

$$\Rightarrow P(E)=\frac{18}{1000}=\frac{9}{500}=0.018\ .$$

(ii) By using Bayes' theorem, $P(E_3\,|\,E)=\dfrac{P(E_3).P(E\,|\,E_3)}{P(E_1).P(E\,|\,E_1)+P(E_2).P(E\,|\,E_2)+P(E_3).P(E\,|\,E_3)}$

$$\Rightarrow P(E_3\,|\,E)=\frac{\frac{7}{10}\times\frac{1}{100}}{\frac{18}{1000}}=\frac{7}{18}.$$

☑ Q.P. Code - 65/1/2

02. If $\vec{\alpha}=\hat{i}-4\hat{j}+9\hat{k}$ and $\vec{\beta}=2\hat{i}-8\hat{j}+\lambda\hat{k}$ are two mutually parallel vectors, then λ is equal to

(a) -18 (b) 18 (c) $-\frac{34}{9}$ (d) $\frac{34}{9}$

Sol. (b) Since given vectors are mutually parallel vectors so, their d.r.'s must be proportional.

That is, $\frac{1}{2}=\frac{-4}{-8}=\frac{9}{\lambda}$.

Considering $\frac{1}{2}=\frac{9}{\lambda} \quad \Rightarrow \lambda = 18$.

03. $\int \frac{1-2\sin x}{\cos^2 x}dx$ is equal to

(a) $\tan x - 2\sec x + C$ (b) $-\tan x + 2\sec x + C$

(c) $-\tan x - 2\sec x + C$ (d) $\tan x + 2\sec x + C$

Sol. (a) $\int \frac{1-2\sin x}{\cos^2 x}dx = \int \left(\sec^2 x - 2\sec x \tan x\right)dx = \tan x - 2\sec x + C$.

06. If A is a square matrix of order 3 such that $\det(A) = 9$, then $\det(9A^{-1})$ is equal to

(a) 9 (b) 9^2 (c) 9^3 (d) 9^4

Sol. (b) $\left|9A^{-1}\right| = 9^3\left|A^{-1}\right| = 9^3\left|A\right|^{-1} = 9^3 \times 9^{-1} = 9^2$.

09. Let A be a matrix of order $m \times n$ and B is matrix such that A^TB and BA^T are defined. Then, the order of B is

(a) $m \times m$ (b) $n \times n$ (c) $m \times n$ (d) $n \times m$

Sol. (c) Let the order of B be $p \times s$. Note that order of A^T will be $n \times m$.

As A^TB is defined so, $p = m$.

Also BA^T is defined so, $s = n$.

Hence, order of B will be $m \times n$.

12. The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$ is

(a) $x e^x$ (b) $\frac{e^x}{x}$ (c) $\frac{x}{e^x}$ (d) $x e^{\frac{1}{x}}$

Sol. (b) Rewriting the D.E., we get $\frac{dy}{dx} + \left(1 - \frac{1}{x}\right)y = \frac{1}{x}$. Here $P(x) = \left(1 - \frac{1}{x}\right)$.

So, integrating factor $= e^{\int\left(1-\frac{1}{x}\right)dx} = e^{x - \log x} = \frac{e^x}{e^{\log x}} = \frac{e^x}{x}$.

13. Let $A = \left[a_{ij}\right]$ be a square matrix of order 3 such that $a_{ij} = j - 2i$. Then which of the following is true?

(a) $a_{12} > 0$ (b) all $a_{ij} < 0$ (c) $a_{13} + a_{31} = -6$ (d) $a_{23} > a_{32}$

Sol. (d) Note that $a_{23} = 3 - 2(2) = -1$ and $a_{32} = 2 - 2(3) = -4$. Clearly, $a_{23} > a_{32}$

Remark In the question, a_{ij} was given as $\hat{j} - 2\hat{i}$, which doesn't seem logical.

22. Find the values of 'a' for which $f(x) = x^2 - 2ax + b$ is an increasing function for $x > 0$.

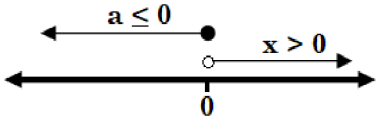
Sol. For $f(x) = x^2 - 2ax + b$, $f'(x) = 2x - 2a$.

Since f is an increasing function so, $f'(x) \geq 0$

That is, $2x - 2a \geq 0$ i.e., $a \leq x$.

As $x > 0$ hence, $a \leq 0$.

Consider the number line shown for clarity.



24. Evaluate $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$.

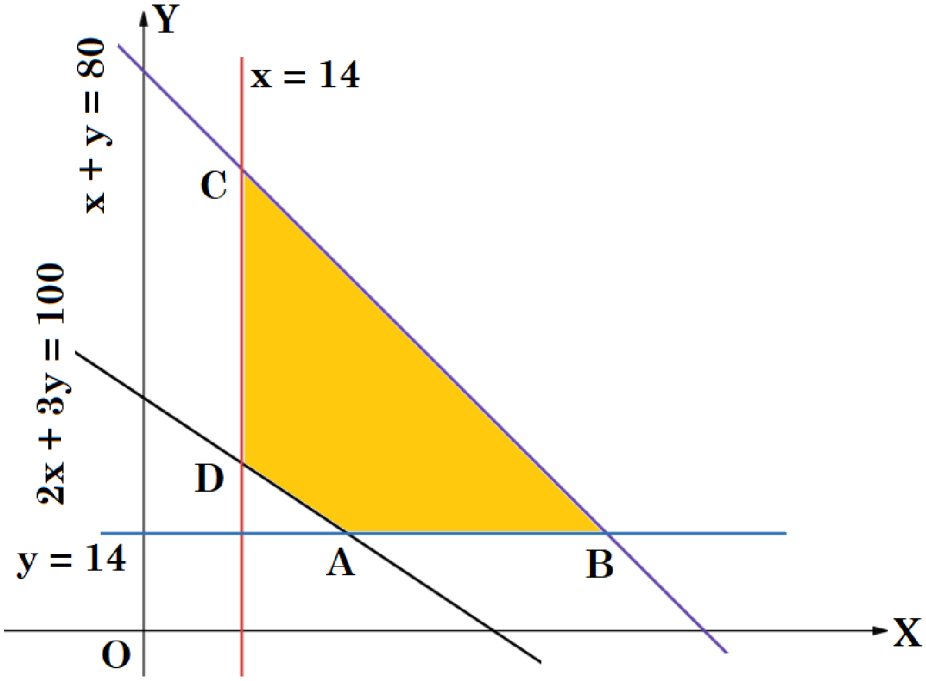
Sol. $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)=\sin^{-1}\left(\sin\left\{\pi-\frac{2\pi}{5}\right\}\right)=\sin^{-1}\left(\sin\frac{2\pi}{5}\right)=\frac{2\pi}{5}$.

27. Solve the following linear programming problem graphically.

Maximise $Z=20x+30y$

Subject to the constraints $x+y\le 80,\ 2x+3y\ge 100,\ x\ge 14,\ y\ge 14$.

Sol. Consider the graph shown below.



The corner points of feasible region are $A(29, 14)$, $B(66, 14)$, $C(14, 66)$ and $D(14, 24)$.

Corner Points	Value of Z
$A(29, 14)$	$Z_A=10(2\times 29+3\times 14)=1000$
$B(66, 14)$	$Z_B=10(2\times 66+3\times 14)=1740$
$C(14, 66)$	$Z_C=10(2\times 14+3\times 66)=2260\leftarrow$ Maximum
$D(14, 24)$	$Z_D=10(2\times 14+3\times 24)=1000$

Hence, the maximum value of Z is 2260 and it is obtained at (14, 66).

28. The area of an expanding rectangle is increasing at the rate of $48\ cm^2/s$. The length of the rectangle is always square of its breadth. At what rate the length of rectangle increasing at an instant, when breadth $= 4.5$ cm ?

Sol. Let the length and breadth of the rectangle be x and y respectively.

It is given that $x=y^2$ …(i)

Now area of rectangle is, $A=xy=x\sqrt{x}=x^{\frac{3}{2}}$ [Using (i)

$$\Rightarrow \frac{dA}{dt}=\frac{3}{2}\sqrt{x}\times\frac{dx}{dt}$$

$$\Rightarrow 48=\frac{3}{2}\times y\times\frac{dx}{dt} \quad \Rightarrow \frac{dx}{dt}=\frac{32}{y}$$

At $y=4.5$ cm, $\frac{dx}{dt}=\frac{32}{4.5}=\frac{320}{45}=\frac{64}{9}$ cm/s.

30. (a) Find $\int \frac{\cos 2x}{(\sin x+\cos x)^2}dx$.

OR

(b) Evaluate $\int_0^{\frac{\pi}{2}} \frac{5\sin x+3\cos x}{\sin x+\cos x}dx$.

Sol. (a) Let $I=\int \frac{\cos 2x}{(\sin x+\cos x)^2}dx=\int \frac{\cos 2x}{\sin^2 x+\cos^2 x+2\sin x\cos x}dx$

$\Rightarrow I=\int \frac{\cos 2x}{1+\sin 2x}dx$ $\left[\text{Put } 1+\sin 2x=t \Rightarrow \cos 2x\,dx=\frac{dt}{2}\right.$

$\Rightarrow I=\frac{1}{2}\int \frac{1}{t}dt=\frac{1}{2}\log|t|+C=\frac{1}{2}\log|1+\sin 2x|+C$.

Also, $I=\frac{1}{2}\log|(\sin x+\cos x)^2|+C=\frac{1}{2}\times 2\log|(\sin x+\cos x)|+C=\log|\sin x+\cos x|+C$.

OR

(b) Let $I=\int_0^{\frac{\pi}{2}} \frac{5\sin x+3\cos x}{\sin x+\cos x}dx$ ...(i)

$\Rightarrow I=\int_0^{\frac{\pi}{2}} \frac{5\sin\left(\frac{\pi}{2}-x\right)+3\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}dx$

$\Rightarrow I=\int_0^{\frac{\pi}{2}} \frac{5\cos x+3\sin x}{\cos x+\sin x}dx$ ...(ii)

Adding (i) and (ii), we get $2I=\int_0^{\frac{\pi}{2}}\left(\frac{5\sin x+3\cos x}{\sin x+\cos x}+\frac{5\cos x+3\sin x}{\cos x+\sin x}\right)dx=\int_0^{\frac{\pi}{2}}8\,dx$

$\Rightarrow I=4[x]_0^{\pi/2}=4\left[\frac{\pi}{2}-0\right]=2\pi$.

32. (a) Differentiate $\tan^{-1}\frac{\sqrt{1-x^2}}{x}$ w.r.t. $\cos^{-1}\left(2x\sqrt{1-x^2}\right)$, $x\in\left(\frac{1}{\sqrt{2}},1\right)$.

OR

(b) Find $\frac{dy}{dx}$, if $y=x^{\tan x}+\frac{\sqrt{x^2+1}}{2}$.

Sol. (a) Let $y=\tan^{-1}\frac{\sqrt{1-x^2}}{x}$ and $z=\cos^{-1}\left(2x\sqrt{1-x^2}\right)$

To simplify, put $x=\cos\theta$ $\Rightarrow \theta=\cos^{-1}x$

As $\frac{1}{\sqrt{2}}<x<1$ i.e., $\frac{1}{\sqrt{2}}<\cos\theta<1$ i.e., $0<\theta<\frac{\pi}{4}$ i.e., $0<2\theta<\frac{\pi}{2}$.

Now $y=\tan^{-1}\frac{\sqrt{1-x^2}}{x}=\tan^{-1}\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}=\tan^{-1}\left(\frac{\sqrt{\sin^2\theta}}{\cos\theta}\right)=\tan^{-1}\left(\frac{|\sin\theta|}{\cos\theta}\right)=\tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)$

$\Rightarrow y = \tan^{-1}(\tan\theta) = \theta \qquad \therefore \frac{dy}{d\theta} = 1 \ \ldots(i)$

And $z = \cos^{-1}\left(2x\sqrt{1-x^2}\right) = \cos^{-1}\left(2\cos\theta\sqrt{1-\cos^2\theta}\right) = \cos^{-1}\left(2\cos\theta|\sin\theta|\right)$

$\Rightarrow z = \cos^{-1}(2\cos\theta\sin\theta) = \cos^{-1}(\sin 2\theta) = \frac{\pi}{2} - \sin^{-1}(\sin 2\theta) = \frac{\pi}{2} - 2\theta \qquad \therefore \frac{dz}{d\theta} = -2 \ \ldots(ii)$

By (i) and (ii), $\frac{dy}{dz} = \frac{dy}{d\theta} \times \frac{d\theta}{dz} = -\frac{1}{2}$.

OR

(b) $y = x^{\tan x} + \frac{\sqrt{x^2+1}}{2}$

$\Rightarrow y = e^{\log x^{\tan x}} + \frac{\sqrt{x^2+1}}{2} = e^{\tan x \log x} + \frac{\sqrt{x^2+1}}{2}$

On differentiating w.r.t. x, we get $\frac{dy}{dx} = e^{\tan x \log x}\left[\tan x \times \frac{1}{x} + \log x \,.\, \sec^2 x\right] + \frac{1}{2} \times \frac{2x}{2\sqrt{x^2+1}}$

$\therefore \frac{dy}{dx} = x^{\tan x}\left[\frac{\tan x}{x} + \log x \,.\, \sec^2 x\right] + \frac{1}{2} \times \frac{x}{\sqrt{x^2+1}}$.

☑ Q.P. Code - 65/1/3

02. The unit vector perpendicular to the vectors $\hat{i} - \hat{j}$ and $\hat{i} + \hat{j}$ is

(a) $\hat{k}$ (b) $-\hat{k} + \hat{j}$ (c) $\frac{\hat{i} - \hat{j}}{\sqrt{2}}$ (d) $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$

Sol. (a) $(\hat{i} - \hat{j}) \times (\hat{i} + \hat{j}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 1 & 1 & 0 \end{vmatrix} = 2\hat{k}$

Required unit vector will be $\hat{k}$.

Note that the unit vector perpendicular to the vectors $\vec{a}$ and $\vec{b}$ both $= \pm\frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$.

03. If $\int_0^1 \frac{e^x}{1+x}dx = \alpha$, then $\int_0^1 \frac{e^x}{(1+x)^2}dx$ is equal to

(a) $\alpha - 1 + \frac{e}{2}$ (b) $\alpha + 1 - \frac{e}{2}$ (c) $\alpha - 1 - \frac{e}{2}$ (d) $\alpha + 1 + \frac{e}{2}$

Sol. (b) Given that $\int_0^1 \frac{e^x}{1+x}dx = \alpha$

$\Rightarrow \left[\frac{e^x}{1+x}\right]_0^1 + \int_0^1 \frac{e^x}{(1+x)^2}dx = \alpha$

$\Rightarrow \left[\frac{e}{2} - 1\right] + \int_0^1 \frac{e^x}{(1+x)^2}dx = \alpha \qquad \therefore \int_0^1 \frac{e^x}{(1+x)^2}dx = \alpha + 1 - \frac{e}{2}$.

Note we have used By Parts method i.e., $\int \frac{e^x}{1+x}dx = \frac{1}{1+x}\int e^x dx - \int\left[\frac{d}{dx}\left(\frac{1}{1+x}\right)\int e^x dx\right]dx$

$$\Rightarrow \int \frac{e^x}{1+x}dx = \frac{e^x}{1+x} + \int\left[\frac{e^x}{(1+x)^2}\right]dx .$$

06. If $\begin{bmatrix} x+y & 3y \\ 3x & x+3 \end{bmatrix} = \begin{bmatrix} 9 & 4x+y \\ x+6 & y \end{bmatrix}$, then $(x-y) =$

(a) -7 (b) -3 (c) 3 (d) 7

Sol. (b) By equality of matrices, we get $x+y=9,\ 3y=4x+y,\ 3x=x+6,\ x+3=y$

On solving the equations, we get $x=3,\ y=6$

Hence, $(x-y)=3-6=-3$.

Alternatively, we can compare *only*, a_{22} i.e., $x+3=y \Rightarrow (x-y)=-3$.

07. Let M and N be two events such that $P(M)=0.6$, $P(N)=0.2$ and $P(M\cap N)=0.5$, then $P(M'\,|\,N')$ is

(a) $\frac{7}{8}$ (b) $\frac{2}{5}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

Sol. (a) $P(M'\,|\,N') = \frac{P(M'\cap N')}{P(N')} = \frac{P(M\cup N)'}{P(N')} = \frac{1-[P(M)+P(N)-P(M\cap N)]}{1-P(N)}$

$$\Rightarrow P(M'\,|\,N') = \frac{1-[0.6+0.2-0.5]}{1-0.2} = \frac{1-0.3}{0.8} = \frac{0.7}{0.8} = \frac{7}{8}.$$

Remark Here $P(M\cap N) > P(N)$, which is not possible.

11. The system of linear equations is represented as $AX=B$, where A is coefficient matrix, X is variable matrix and B is the constant matrix. Then the system of equations

(a) is consistent, if $|A|\neq 0$, solution is given by $X = BA^{-1}$

(b) is inconsistent, if $|A|=0$ and $(\text{adj}\,A)B=O$

(c) is inconsistent, if $|A|\neq 0$

(d) may or may not be consistent, if $|A|=0$ and $(\text{adj}\,A)B=O$

Sol. (d) may or may not be consistent, if $|A|=0$ and $(\text{adj}\,A)B=O$.

13. The order and degree of differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^5 = \frac{d^2y}{dx^2}$ are

(a) order 1, degree 1 (b) order 1, degree 2

(c) order 2, degree 1 (d) order 2, degree 2

Sol. (c) order 2, degree 1.

22. Find the values of 'a' for which $f(x)=\sqrt{3}\sin x - \cos x - 2ax + b$ is decreasing on $\mathbb{R}$.

Sol. For $f(x)=\sqrt{3}\sin x - \cos x - 2ax + b$, $f'(x)=\sqrt{3}\cos x + \sin x - 2a$

$$\Rightarrow f'(x) = 2\left(\frac{\sqrt{3}}{2}\cos x + \sin x \times \frac{1}{2}\right) - 2a = 2\left(\sin\frac{\pi}{3}\cos x + \sin x\cos\frac{\pi}{3}\right) - 2a$$

$$\Rightarrow f'(x) = 2\sin\left(\frac{\pi}{3}+x\right) - 2a$$

As $f(x)$ is decreasing on $\mathbb{R}$ so, $f'(x)\leq 0$ i.e., $2\sin\left(\frac{\pi}{3}+x\right) - 2a \leq 0$

$$\therefore a \geq \sin\left(\frac{\pi}{3}+x\right) \ldots(i)$$

We know that for all $x \in \mathbb{R}$, $\sin\left(\frac{\pi}{3}+x\right) \in [-1, 1]$ i.e., $-1 \le \sin\left(\frac{\pi}{3}+x\right) \le 1$ ...(ii)

By (i) and (ii), we conclude that $a \ge 1$.

24. Solve for x : $2\tan^{-1} x + \sin^{-1}\left(\frac{2x}{1+x^2}\right) = 4\sqrt{3}$.

Sol. Let $x = \tan\theta \Rightarrow \theta = \tan^{-1} x$.

So, $2\tan^{-1} x + \sin^{-1}\left(\frac{2x}{1+x^2}\right) = 4\sqrt{3}$ becomes $2\theta + \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = 4\sqrt{3}$

$\Rightarrow 2\theta + \sin^{-1}(\sin 2\theta) = 4\sqrt{3}$

$\Rightarrow 2\theta + 2\theta = 4\sqrt{3} \quad \Rightarrow 4\theta = 4\sqrt{3}$

$\Rightarrow \tan^{-1} x = \sqrt{3} \notin \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. So, the given equation has **no** solution.

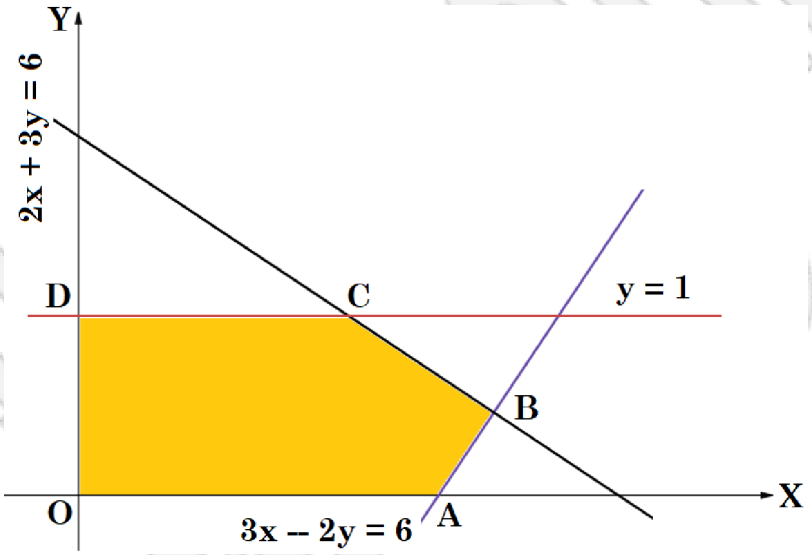
Note that, $\tan^{-1} x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\sqrt{3} \notin \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. That is, $x = \phi$.

26. Solve the following linear programming problem graphically.

Maximize $Z = 8x + 9y$

Subject to the constraints $2x + 3y \le 6$, $3x - 2y \le 6$, $y \le 1$, $x \ge 0$, $y \ge 0$.

Sol. Consider the graph shown below.



The corner points of feasible region are $O(0, 0)$, $A(2, 0)$, $B\left(\frac{30}{13}, \frac{6}{13}\right)$, $C\left(\frac{3}{2}, 1\right)$ and $D(0, 1)$.

Corner Points	Value of Z
$O(0, 0)$	$Z_O = 8\times 0 + 9\times 0 = 0$
$A(2, 0)$	$Z_A = 8\times 2 + 9\times 0 = 16$
$B\left(\frac{30}{13}, \frac{6}{13}\right)$	$Z_B = 8\times\frac{30}{13} + 9\times\frac{6}{13} = \frac{294}{13}$ ← Maximum
$C\left(\frac{3}{2}, 1\right)$	$Z_C = 8\times\frac{3}{2} + 9\times 1 = 21$
$D(0, 1)$	$Z_D = 8\times 0 + 9\times 1 = 9$

Hence, the maximum value of Z is $\frac{294}{13}$ and it is obtained at $\left(\frac{30}{13}, \frac{6}{13}\right)$.

27. (a) Find $\int \frac{2x-1}{(x-1)(x+2)(x-3)}dx$.

OR

(b) Evaluate $\int_0^5 \left(|x-1|+|x-2|+|x-5|\right)dx$.

Sol. (a) Consider $\frac{2x-1}{(x-1)(x+2)(x-3)} = \frac{A}{x-1}+\frac{B}{x+2}+\frac{C}{x-3}$

$\Rightarrow 2x-1 = A(x+2)(x-3)+B(x-1)(x-3)+C(x-1)(x+2)$

On comparing the coefficients of like terms on both the sides, we get $A = -\frac{1}{6}$, $B = -\frac{1}{3}$, $C = \frac{1}{2}$.

So, $\int \frac{2x-1}{(x-1)(x+2)(x-3)}dx = -\frac{1}{6}\int \frac{1}{x-1}dx - \frac{1}{3}\int \frac{1}{x+2}dx + \frac{1}{2}\int \frac{1}{x-3}dx$

$= -\frac{1}{6}\log|x-1| - \frac{1}{3}\log|x+2| + \frac{1}{2}\log|x-3| + C$.

OR

(b) $\int_0^5 \left(|x-1|+|x-2|+|x-5|\right)dx = \int_0^5 |x-1|dx + \int_0^5 |x-2|dx + \int_0^5 |x-5|dx$

$= \int_0^1 |x-1|dx + \int_1^5 |x-1|dx + \int_0^2 |x-2|dx + \int_2^5 |x-2|dx + \int_0^5 -(x-5)dx$

$= \int_0^1 -(x-1)dx + \int_1^5 (x-1)dx + \int_0^2 -(x-2)dx + \int_2^5 (x-2)dx + \int_0^5 -(x-5)dx$

$= -\left[\frac{(x-1)^2}{2}\right]_0^1 + \left[\frac{(x-1)^2}{2}\right]_1^5 - \left[\frac{(x-2)^2}{2}\right]_0^2 + \left[\frac{(x-2)^2}{2}\right]_2^5 - \left[\frac{(x-5)^2}{2}\right]_0^5$

$= -\left\{0-\frac{1}{2}\right\} + \{8-0\} - \{0-2\} + \left\{\frac{9}{2}-0\right\} - \left\{0-\frac{25}{2}\right\}$

$= \frac{1}{2}+10+\frac{9}{2}+\frac{25}{2} = \frac{35}{2}+10 = \frac{55}{2}$.

28. A spherical medicine ball when dropped in water dissolves in such a way that the rate of decrease of volume at any instant is proportional to its surface area. Calculate the rate of decrease of its radius.

Sol. Let 'r' be the radius.

Now $-\frac{dV}{dt} \propto S$, where S is the surface area

$\Rightarrow \frac{dV}{dt} = -kS$, where k is a positive proportionality constant

$\Rightarrow \frac{d}{dt}\left[\frac{4}{3}\pi r^3\right] = -k \times 4\pi r^2$

$\Rightarrow \frac{4}{3}\pi \times 3r^2 \frac{dr}{dt} = -k \times 4\pi r^2$

$\Rightarrow \frac{dr}{dt} = -k$ i.e., radius decreases at a constant rate.

33. Find $\int \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx$.

Sol. Let $I = \int \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx$ (Put $x = a\tan^2\theta \Rightarrow dx = 2a\tan\theta\sec^2\theta\,d\theta$

$\therefore I = \int \sin^{-1}\sqrt{\frac{a\tan^2\theta}{a+a\tan^2\theta}}\,2a\tan\theta\sec^2\theta\,d\theta$

$\Rightarrow I = 2a\int \sin^{-1}(\sin\theta)\tan\theta\sec^2\theta\,d\theta$

$\Rightarrow I = 2a\int \theta(\tan\theta\sec^2\theta)\,d\theta$

$\Rightarrow I = 2a\left\{\theta\int \tan\theta\sec^2\theta\,d\theta - \int\left(\frac{d}{d\theta}(\theta)\int \tan\theta\sec^2\theta\,d\theta\right)\right\}$

$\Rightarrow I = 2a\left\{\theta\frac{\tan^2\theta}{2} - \int\frac{\tan^2\theta}{2}d\theta\right\}$

$\Rightarrow I = a\left\{\theta\tan^2\theta - \int(\sec^2\theta - 1)\,d\theta\right\}$

$\Rightarrow I = a\left\{\theta\tan^2\theta - (\tan\theta - \theta)\right\} + C$

$\Rightarrow I = a\left\{\frac{x}{a}\tan^{-1}\left(\sqrt{\frac{x}{a}}\right) - \sqrt{\frac{x}{a}} + \tan^{-1}\left(\sqrt{\frac{x}{a}}\right)\right\} + C$

$\therefore I = (x+a)\tan^{-1}\sqrt{\frac{x}{a}} - \sqrt{ax} + C$.

With regards,
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